

# Generating dark solitons by single photons

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We show that dark solitons in 1D bose systems may be excited by resonant absorption of single quanta of an external ac field. The field frequency  $\omega$  should be slightly blue-detuned from  $\varepsilon_s(\hbar q)/\hbar$ , where  $\varepsilon_s(\hbar q)$  is the energy of a soliton with momentum corresponding to the external field wavenumber  $q$ . We calculate the absorption cross-section and show that it has power-law dependence on the frequency detuning  $\omega - \varepsilon_s(\hbar q)/\hbar$ . This reflects the quantum nature of the absorption process and the orthogonality catastrophe phenomenon associated with it.

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The existence of dark solitons (DS) is probably the most spectacular manifestation of the role played by *weak* inter-particle interactions in 1D cold atomic gases [1]. Such solitons are *macroscopically* large areas of partially, or even completely depleted gas, which propagate without dispersion. It is natural to interpret these objects as solutions of the *classical* Gross–Pitaevskii equation [2, 3]. Correspondingly, the means to create DS, employed so far, required a macroscopic classical perturbation applied to the atomic cloud. An example of the latter is the phase imprinting technique [4], where a finite fraction of the 1D cloud is subject to an external potential for a certain time.

Yet, there is a deep connection between DS and intrinsically *quantum* nature of the gas. It was first appreciated by Kulish, Manakov and Faddeev [5], who noticed that the soliton’s dispersion relation coincides [6] with the exact lower bound of the *quantum many-body* spectrum of the Lieb–Liniger model [7], Fig. 1. The existence of such a bound, known also as the Lieb II mode, was derived earlier using Bethe Ansatz technique [7]. This observation means that the DS may be considered as a classical approximation of very peculiar quantum many-body eigenstates: those which possess the minimal possible energy  $\varepsilon_2$  at a given momentum  $\hbar q$ , see Fig. 1.

In this Letter we show that the underlying quantum nature of the gas allows for a qualitatively different way of creating DS. Namely, the soliton may be generated by the absorption of a *single quantum* [8] of an ac field slightly blue-detuned from the resonance with the soliton energy  $\hbar\omega \gtrsim \varepsilon_2(\hbar q)$ , where  $\omega$  and  $q$  are frequency and wavenumber of the ac field. Notice, that the comparison between the soliton’s energy and the ac frequency does not appear in the classical treatment at all.

The quantum efficiency of the discussed process is limited by the orthogonality catastrophe [9]. Indeed, a state of the system after it is excited from the ground state by absorption of one quantum of the ac field has a very small overlap with the eigenstates forming the soliton. As a result, the quantum yield exhibits the very characteristic power-law dependence on the energy excess above the

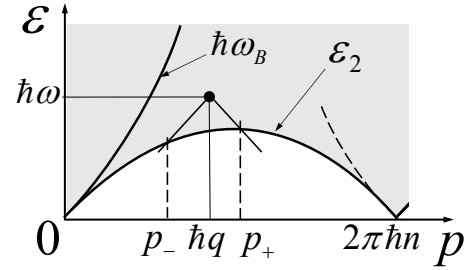


FIG. 1: Energy vs momentum plane for excitations. A photon is represented by a solid dot. The “light cone” with Bogoliubov velocity  $v_B$  determines the range of possible momenta of generated dark solitons,  $p_- < p_s < p_+$ .

threshold:  $\sim (\hbar\omega - \varepsilon_2)^{\mu_2}$ . The corresponding exponent  $\mu_2$  is a function of the wavenumber  $q$  and the dimensionless interaction parameter  $\gamma$ . For weak interactions,  $\gamma \ll 1$ , the exponent is large,  $\mu_2 \sim \gamma^{-1/2} \gg 1$ , reflecting the strong orthogonality. The photon absorption and DS production, associated with it, are higher for a moderate interaction parameter,  $\gamma \lesssim 1$ , when the DS density depletion is weaker.

To set the notation let us briefly remind what the localized solutions of the non-linear Gross–Pitaevskii equation are. Quasiclassically, the condensate wave function obeys the following equation [10]:

$$i\partial_t\Psi + \frac{\hbar}{2m}\partial_x^2\Psi + c(n - |\Psi|^2)\Psi = 0, \quad (1)$$

where  $n = N/L$  is the average concentration and  $L$  is the length of the system with the periodic boundary conditions. The interaction strength  $c$  gives rise [7] to the dimensionless parameter  $\gamma = cm/(\hbar n)$  whose smallness  $\gamma \ll 1$  is the criterion of the weakly interacting gas.

Soliton solutions of Eq. (1) have the form [11]

$$\Psi_s = \sqrt{n} \left[ \cos \frac{\theta_s}{2} - i \sin \frac{\theta_s}{2} \tanh \left( \frac{x - v_s t}{l_s} \right) \right] e^{i \frac{2\pi - L}{2L} \theta_s}, \quad (2)$$

where the only free parameter  $\theta_s$  is the change of phase of the condensate wave function  $\Psi_s = \sqrt{n(x)}e^{i\vartheta(x)}$  across the soliton. The condensate density  $n(x)$  reaches equilibrium value  $n$  away from the soliton (see Fig. 2). For a given  $\theta_s$  the soliton velocity and its spatial extent are fixed and given by  $v_s = v_B \cos(\theta_s/2)$  and  $l_s = \hbar(mv_B \sin(\theta_s/2))^{-1}$ , where  $v_B$  denotes the Bogoliubov sound velocity, which in the weakly interacting gas is given by  $v_B = \sqrt{\gamma} \hbar n/m$ . The soliton, Eq. (2), represents the density depletion, propagating without dispersion with the velocity  $v_s$ . The number of particles pushed away from the soliton core is

$$N_s = \frac{2K}{\pi} \sin \frac{\theta_s}{2}, \quad (3)$$

where  $K = \pi \hbar n/(mv_B)$  is the thermodynamic compressibility of the gas.

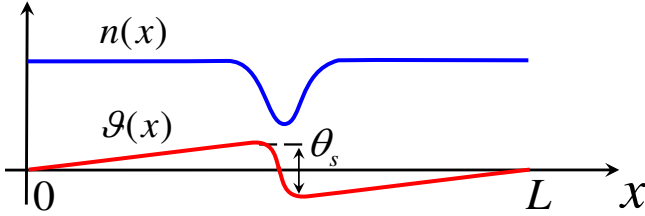


FIG. 2: (color online) Density  $n(x)$  and phase  $\vartheta(x)$  profiles of a soliton in a system of length  $L$ , as given by Eq. (2). Note that the density perturbation is local, while the perturbation of phase is not.

In the presence of a soliton the system acquires a non-zero momentum  $p_s$  and energy  $\varepsilon_s$  which are expressed through the phase shift  $\theta_s$  as:

$$p_s = \hbar n (\theta_s - \sin \theta_s); \quad \varepsilon_s = \frac{4}{3} \hbar n v_B \sin^3 \frac{\theta_s}{2}. \quad (4)$$

These two relations implicitly define the soliton dispersion relation  $\varepsilon_s = \varepsilon_s(p_s)$ . One may check that the velocity of the soliton satisfies  $v_s = \partial \varepsilon_s / \partial p_s$ , as expected for a particle. It is important to stress that while the energy  $\varepsilon_s$  is entirely associated with the soliton core, the momentum  $p_s$  is shared between the core and the rest of the gas. More precisely, the soliton core carries the momentum  $-\hbar n \sin \theta_s$  (the negative sign here corresponds to a hole in the density), whereas the momentum  $\hbar n \theta_s$  is uniformly spread to the rest of the gas. The latter fact is easily seen from the phase  $\vartheta(x)$  profile, Fig. 2, and the observation that the momentum density of the condensate is given by  $\hbar n \partial_x \vartheta$ .

At small momenta  $p_s \ll \pi \hbar n$ , one finds  $\theta_s \rightarrow 0$  and thus  $\varepsilon_s \approx v_B p_s$ , cf. Eq. (4). In this limit the soliton dispersion approaches the phonon branch [12] characterized by the Bogoliubov spectrum  $\omega_B(q) = (v_B q) \sqrt{1 + (\hbar q / 2 m v_B)^2}$ , see Fig. 1. On the other hand, for  $p_s = \pi \hbar n$ , the phase shift is  $\theta_s = \pi$  so the soliton is

at rest:  $v_s = 0$ . In the latter case the entire momentum  $p_s$  is uniformly spread over the bulk of the gas.

Imagine now that the gas is subject to a weak space and time dependent external potential  $V_0 \cos(qx - \omega t)$ . According to the Fermi Golden rule, the system may absorb quanta of this field if its many-body spectrum possesses excited states with the momentum  $\hbar q$  and energy  $\hbar \omega$ . As may be learned from the exactly solvable model [7], such states form a continuum whose energy is bound from below by the line  $\varepsilon_2(\hbar q)$  (so called Lieb II mode). It was subsequently noticed in Ref. [5] that in the limit of weak interactions  $\gamma \ll 1$  the Lieb II mode coincides with the soliton dispersion relation, i.e.  $\varepsilon_2(\hbar q) = \varepsilon_s(\hbar q)$ . These observations imply that an absorption of a quantum blue-detuned from the soliton energy  $\hbar \omega \gtrsim \varepsilon_s(\hbar q)$  is (i) possible and (ii) leads to creation of the soliton. Our aim is to study the scattering crosssection of such processes.

To this end we notice that a photon absorption first creates a *virtual* state of the condensate with a *local* perturbation of the condensate wavefunction. Since the photon carries momentum  $\hbar q$  and carries no extra particles, so does the initial local perturbation. Subsequently this perturbation evolves according to the equations of motion and eventually it must take a form of a superposition of *real* excitations, i.e. conserving overall energy  $\hbar \omega$  in addition to the momentum  $\hbar q$ . We expect that such a final state contains a soliton with momentum  $p_s \approx \hbar q$  and core energy  $\varepsilon_s(p_s) < \hbar \omega$ . The small excess energy  $\hbar \omega - \varepsilon_s > 0$  and the momentum difference  $\hbar q - p_s$  are carried away by the small-amplitude sound waves (phonons) with velocity  $v_B$ . This observation immediately implies that  $v_B |\hbar q - p_s| \leq (\hbar \omega - \varepsilon_s(p_s))$ , with the equality reached if all the sound waves are emitted in one direction only. As a result, the range  $p_- \leq p_s \leq p_+$  of possible soliton momenta  $p_s$  is limited

$$p_{\pm}(q, \omega) \approx \hbar q \pm \frac{\hbar \omega - \varepsilon_s(\hbar q)}{v_B \pm v_s(\hbar q)}; \quad p_+ - p_- \ll p_s. \quad (5)$$

The corresponding construction is shown in Fig. 1.

We found that the inelastic scattering crosssection for absorbing a photon with the wavenumber  $q$  and frequency  $\omega$ , while creating the dark soliton with the momentum  $p_s$  is given by

$$W_{q,\omega}(p_s) \sim \frac{l_s}{\hbar v_B} \left[ \frac{p_+ - p_s}{\hbar/l_s} \right]^{\mu_+ - 1} \left[ \frac{p_s - p_-}{\hbar/l_s} \right]^{\mu_- - 1}. \quad (6)$$

It is characterized by the power-law dependencies on the deviations of the soliton momentum from the upper and lower kinematic boundaries  $p_{\pm}(q, \omega)$ . The corresponding exponents  $\mu_{\pm}$  are functions of the soliton parameter  $\theta_s = \theta_s(q)$  and the thermodynamic compressibility  $K$ ,

$$\mu_{\pm}(q) = \frac{K}{4} \left( \frac{\theta_s}{\pi} \pm \frac{N_s}{K} \right)^2 = \frac{K}{\pi^2} \left( \frac{\theta_s}{2} \pm \sin \frac{\theta_s}{2} \right)^2. \quad (7)$$

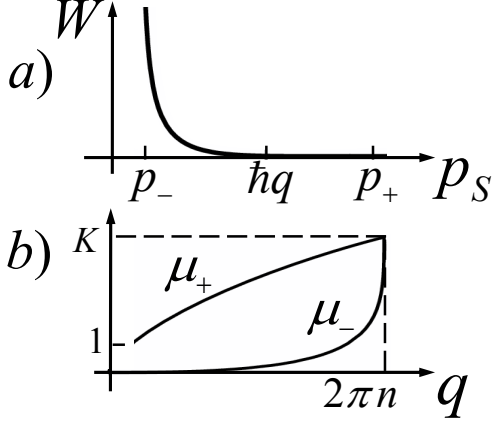


FIG. 3: (a) Inelastic scattering crosssection as a function of the soliton momentum  $p_s$ . (b) Momentum dependence of the exponents  $\mu_{\pm}$  for  $K = 10$ . The exponent in Eq. (8) is given by  $\mu_2(q) = \mu_+(q) + \mu_-(q) - 1$ .

In the last equality here we employed Eq. (3), and the function  $\theta_s(q)$  is implicitly defined by Eq. (4) with  $p_s = \hbar q$ . The calculated crosssection as a function of the soliton momentum  $p_s$  for a photon with  $q = \pi n$  is plotted in Fig. 3a. We have also plotted the  $q$ -dependence of exponents  $\mu_{\pm}(q)$  in Fig. 3b.

Equations (5) – (7) are the main result of this paper. They provide the probability of the soliton excitation with a specific momentum  $p_s$  (and therefore specific velocity  $v_s = \partial \varepsilon_s / \partial p_s$ ). One may also be interested in the integral probability of absorbing a photon with a given  $q$  and  $\omega \gtrsim \varepsilon_s(\hbar q) / \hbar$ , which results in creation of a soliton with an unspecified momentum. This quantity is nothing but the dynamic structure factor (DSF)  $S(q, \omega)$  of the 1D Bose gas. Integrating  $W_{q, \omega}(p_s)$ , Eq. (6), over the soliton momenta  $p_s$ , one finds for DSF in an immediate vicinity of the lower spectral boundary  $\hbar \omega \gtrsim \varepsilon_2(\hbar q)$

$$S(q, \omega) \sim \frac{1}{v_B} \left[ \frac{\hbar \omega - \varepsilon_2(\hbar q)}{\hbar v_B / l_s} \right]^{\mu_2(q)} \theta(\hbar \omega - \varepsilon_2). \quad (8)$$

Being multiplied by the intensity of the radiation  $V_0^2$  this quantity gives a number of solitons excited per unit time and per unit length of the irradiated 1D gas. According to Eq. (7), the wavenumber-dependent exponent  $\mu_2(q) = \mu_+(q) + \mu_-(q) - 1$  is given by

$$\mu_2(q) = \frac{2K}{\pi^2} \left[ \left( \frac{\theta_s}{2} \right)^2 + \left( \sin \frac{\theta_s}{2} \right)^2 \right] - 1, \quad (9)$$

The power law behavior of DSF near the lower spectral boundary  $\varepsilon_2(\hbar q)$  was suggested earlier in Ref. [13]. However, Ref. [13] derived functional dependence of  $\mu_2(q)$  only in the limit of strong interactions,  $\gamma \gg 1$ . Equation (9) provides the answer in the opposite limit of weak scattering,  $\gamma \ll 1$ .

Below we outline the details of derivation. As was explained earlier, the initial *local* perturbation of the condensate carries the momentum of the absorbed photon  $\hbar q$  and no excess particles. After a short time, which may be estimated as  $\tau_s = l_s / v_B$ , the soliton core is formed along with a bunch of outgoing sound waves. The core, which is the density depletion, carries momentum  $-\hbar n \sin \theta_s$  and  $-N_s$  particles. At times  $t > \tau_s$  it propagates without dispersion and behaves as a free particle with the energy  $\varepsilon_s$ . The remaining momentum  $\hbar(q + n \sin \theta_s) = \hbar n \theta_s$ , cf. Eq. (4), and  $N_s$  particles, localized on a scale  $\sim l_s$  at  $t \sim \tau_s$  must be carried away and spread over the entire system at  $t \gg \tau_s$  by the linear *sound waves*. The latter are conventionally described [14] by the linearized hydrodynamic Hamiltonian

$$H_{sw} = \frac{\hbar v_B}{2\pi} \int dx \left[ K^{-1} (\partial_x \hat{\varphi})^2 + K (\partial_x \hat{\vartheta})^2 \right], \quad (10)$$

where  $\hbar n \partial_x \hat{\vartheta}(x)$  and  $\partial_x \hat{\varphi} / \pi$  are operators of momentum and particle excess densities, correspondingly. Their canonical commutation relation reads  $[\hat{\varphi}(x), \hat{\vartheta}(y)] = i(\pi/2) \text{sgn}(x - y)$ .

The injection of momentum  $\hbar n \theta_s$  and  $N_s$  particles into the sound waves at a point  $x$  is achieved by acting with operator

$$\hat{\psi}_{sw}^\dagger(x) = e^{i \frac{\theta_s}{\pi} \hat{\varphi}(x) + i N_s \hat{\vartheta}(x)} \quad (11)$$

on the ground state of the Hamiltonian (10). Indeed, the operator  $e^{i \frac{\theta_s}{\pi} \hat{\varphi}(x)}$  shifts  $\hat{\vartheta}(y)$  by  $\theta_s$  at  $x > y$ , accommodating momentum  $\hbar n \theta_s$ . Similarly,  $e^{i N_s \hat{\vartheta}(x)}$  shifts  $\hat{\varphi}(y)$  by  $N_s$ , at  $x > y$  accommodating  $N_s$  particles at the point  $x$ . Since we are dealing with large shifts, we can disregard non-commutativity of the operators, effectively adopting the semiclassical approximation (see below).

The energy of a state created immediately upon the shift Eq. (11) is much higher than  $\hbar \omega(q) - \varepsilon_s(\hbar q)$ . In full analogy with the instanton picture of tunneling across a weak link in a Luttinger liquid [15], the created virtual state evolves, eventually reducing its energy to  $\hbar \omega(q) - \varepsilon_s(\hbar q)$ . To find the photon absorption crosssection resulting in a creation of a dark soliton, one needs to evaluate the time evolution of the density–density correlator of the excited sound waves,

$$G(x, t) = \langle \hat{\psi}_{sw}(x, t) \hat{\psi}_{sw}^\dagger(0, 0) \rangle. \quad (12)$$

Its calculation uses Eqs. (11) and (10) and follows the standard route of the bosonization theory [14], leading to

$$G(x, t) = \left[ 1 + \frac{x - v_B t}{i l_s} \right]^{-\mu_+} \times \left[ 1 - \frac{x + v_B t}{i l_s} \right]^{-\mu_-}. \quad (13)$$

Here the exponents are given by Eq. (7) and  $l_s$  enters through the short-time cutoff  $\tau_s$  as  $l_s = v_B \tau_s$ . Finally

the absorption crosssection is proportional to the Fourier transform of the correlator:

$$W_{q,\omega}(p_s) \sim \text{Im} \int dx dt G(x, t) e^{i(\omega - \varepsilon_s/\hbar)t - (q - p_s/\hbar)x}, \quad (14)$$

where we took into account the momentum  $p_s$  and energy  $\varepsilon_s(p_s)$  carried away by the soliton. To evaluate the integral in Eq. (14), we take into account that for small energy excess  $\hbar\omega - \varepsilon_s(\hbar q) \ll \hbar\omega$  the range of the allowed soliton momenta  $p_s$  is rather narrow, see Fig. 1 and Eq. (5), and centered around the photon momentum  $\hbar q$ . One may therefore expand the soliton energy as  $\varepsilon_s(p_s) \approx \varepsilon_s(\hbar q) + (p_s - \hbar q)v_s$ . Equations (5) and (6) then follow from the straightforward calculation.

The developed theory is applicable in the vicinity of the spectrum of the Lieb II mode. The width of the corresponding region in the  $(q, \omega)$  plane, see Fig. 1, is determined by the condition  $\hbar\omega - \varepsilon_2(\hbar q) \lesssim \hbar/\tau_s$ . Also  $\omega$  is restricted to be below the Bogoliubov mode  $\omega_B$  and its replica [18], shown by the dashed line in Fig. 1. The soliton production rate reaches its maximum at  $\hbar\omega - \varepsilon_2(\hbar q) \sim \hbar/\tau_s$  and decreases at higher frequency.

Additional restrictions are set by the applicability of semiclassical approximation. The semiclassical description of the Lieb II mode by dark solitons fails at  $p$  sufficiently close to 0 or  $2\pi\hbar n$ . This already can be seen from a comparison [16] of the true spectrum of the Lieb II mode with the prediction of the semiclassical Gross-Pitaevskii approach. At  $\gamma \ll 1$  the two spectra significantly differ from one another at  $p \lesssim \hbar n \gamma^{3/4}$ . Equations (7) and (9) predict monotonic decrease of the exponents  $\mu_{\pm}$  and  $\mu_2$  with the decrease of  $q$ . For  $q \ll \pi n$  they yield

$$\mu_2(q) \approx \mu_+(q) = \frac{K}{\pi^2} \left( \frac{6q}{n} \right)^{2/3}, \quad \mu_- \propto q^2 \ll \mu_+. \quad (15)$$

This behavior is actually valid in the interval  $n\gamma^{3/4} \ll q \ll \pi n$ , the lower limit here being set by the applicability of the semiclassical approximation. In this range the exponents  $\mu_2, \mu_+ \gg 1$ , while there are no further restrictions on  $\mu_- > 0$ .

Note that in the case of strong interaction [13] ( $\gamma \gg 1$ ), the asymptote at  $q \rightarrow 0$  is  $\mu_2(q) \propto q$ . In this limit DS degenerates into a single hole within weakly-interacting *Fermi sea* in the effective fermionic description [17]. We expect the linearity must stay for the smallest  $q$  at any interaction strength, and thus expect a crossover from  $\mu_+, \mu_2 \propto q^{2/3}$  to the linear behavior at  $q \lesssim n\gamma^{3/4}$ . The similar crossover occurs in the narrow vicinity of the  $q = 2\pi n$  point, where the semiclassical soliton description also runs out of the applicability.

To conclude, we have shown that DS may be generated by absorption of a single photon of an external ac field. Quantum efficiency of such a process is maximized at slight,  $\sim \hbar/\tau_s$ , blue detuning of the photon energy  $\hbar\omega$

from the soliton energy  $\varepsilon_s(\hbar q)$ . Within this range the absorption probability behaves as the power law of the detuning, reflecting the quantum orthogonality catastrophe phenomenon.

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